Describing fields

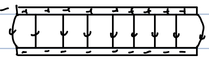
- Formula:
  - Work - electric potential equation: $W = Q \Delta V_e$
  - Work - gravitational potential equation: $W = m \Delta V_g$
- CHAPTER COMMENT: Electric fields: blue highlight, while gravity fields are given by a green highlight.
- Fields
  - A field is said to exist when one object can exert a force on another object at a distance.
  -

Electric fields	Gravity Fields
An electrostatic force exists between two charged objects	A gravitational force exists between two objects that both have mass.
There are two types of charge: - negative = surplus in electrons - positive = deficit of electrons Same charge = repulsion Opposite charge = attraction	A gravitational field is associated with each mass. Any other mass in this field has a gravitational field acting on it.
An electric charge leads to an electric field. An electrostatic force acts on a charge that's in the field of another charge.	Gravity is always an attractive force. Repulsion between masses is never observed.

- Field strength
  - Electric field strength = $\frac{\text{force acting on positive test charge}}{\text{magnitude of test charge}}$ → $F = \frac{k q_1 q_2}{r^2}$

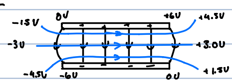
$NC^{-1} = \frac{F}{q}$

$E = \frac{kq}{r^2}$

  - The direction of the field is the same as the direction of the force acting on a positive charge.
  - Units = NC^{-1}
  - Gravitational field strength = $\frac{\text{force acting on test mass}}{\text{magnitude of test mass}}$
    - Gravitational force is always attractive.
    - Direction of field strength is always the mass that gives rise to the field strength.
    - Nkg^{-1}.
- Energy ideas so far
  - Electric fields
    - Electric pd: $V = \frac{W}{Q}$
      - V = electric pd, W = work done, and Q is charge of a positive charge.
      - $V = JC^{-1}$
  - Gravity Fields (Potential energy)
    - $\Delta E_p = mg\Delta h$
      - ΔE_p is change in gravitational potential energy, m = mass, g = gravity, and Δh = change in height.
      - ΔE_p = Joules
- Field lines
  - If a charged particle/mass passes through an electric field with relevant field property (mass for gravity, charge for electricity) it'll be affected & it'll move away from the side with the same charge.
  - The fields between two parallel are:
    - Uniform in the region between the plates.
    - Becomes weaker at the edges (known as edge effects) as the field changes from the value between the plates value to the outside-plates value.
  - 
    - The weaker lines (curved lines) show how the field gradually weakens.
  - Lines of forces is a line in a field of force whose tangent at any point gives the direction of the field at that point.

- The lines of force originate at positive charge (by definition) and end at the negative charged side.

- The lines of equal potential difference between two parallel plates:

  - 

Why are some sides negative, and the other positive?
The reason being that the field lines are pointing to the right which means that the opposite direction, left, will be negative as its a vector quantity.

Black lines = electric field lines, Blue lines = equipotentials

  - These lines are called equipotentials, as they show points where the voltage is always the same.
    - This means that when a charge moves from one point to another along an equipotential line, no work is done.

- There is a simple relationship between electrical field lines and equipotentials. The relationship is that the angle between them will always be 90°.

- The values for "potential" are the same as "potential difference".
  - Potential differences are measured final state minus initial state. E.g: final state = +1.5 V, initial = -3.0 V ΔV = +1.5V.

- The figure 4 shows the potential lines and electric lines in 2 dimensions rather than 3 dimensions.
  - In 3 dimensions the rule of electric field always being 90° to the potential lines will remain true.
  - Therefore in three dimensions its possible to have equipotential surfaces or volumes.

- An example of equipotential surfaces or volumes is solid conductors.
  - In a solid conductor if there is a potential difference between any part of the conductor and another, then charge will flow until the potential difference is zero.
  - All parts of the conductor must therefore be at the same potential as each other, meaning the whole conductor is an equipotential volume. Therefore, the field lines must exit the volume at 90°, no matter the shape.

- Summary of equipotential surfaces or volumes:
  - Link points have the same potential
  - They are regions where charge can move without work being done on or by the charge.
  - Field lines are at 90°.
  - Have zero potential.
  - Don't have direction (potential, like any energy, is a scalar quantity).
  - Can't ever meet or cross another equipotential that has a different value.

- Equipotential and field lines in other situations

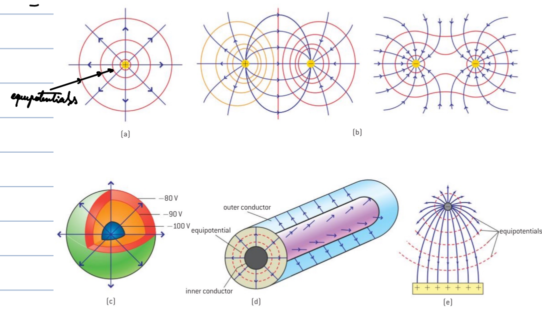
- Equipotentials and electric fields lines for a number of common situations.
  - 

Figure 6 Equipotentials around charge arrangements.

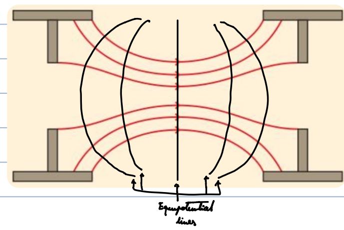
- Field lines due to a single point charge
  - For a single positive charge, the field lines radiate out from the point and the field is called a radial field.
  - For a single negative charge the field lines are the same, but instead of pointing outwards they point inwards.
  - The equipotentials for figure 1a are the shells around the point charge.
    - While they're in 2D, its necessary to think of them in 3D.

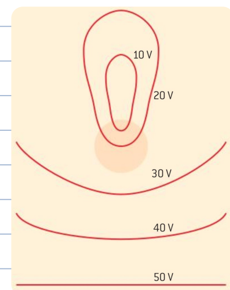
- Field lines due to two points charges of the same and opposite sign
  - Seen in figure 6.
  - Have some similarities to the magnetic patterns in the space between two bar magnets.

- Field due to a charged sphere (figure c)
  - A hollow or solid conducting sphere is at a single potential.
  - The field outside of the sphere is identical to the point charge of the same magnitude.
    - Although, there are no field lines in the sphere.

- Field due to a co-axial conductor (figure d)
  - These conductors have a central conductor with an earthed cylinder around it, separated and spaced by an insulator.

- The symmetry of the arrangement is different from that of a sphere and so the symmetry of field patterns changes as well.
- Field between a point charge and a charged plate (figure 2)
    - The field lines are radial to the point charge when very close to it and the lines are 90° to the surface of the plane.
- Field and equipotential in gravitation
    - The arrangement of gravitational field lines that surround a point mass or a spherical planet is similar to the pattern for electric field lines around a negative point charge, the reason being that the gravitational field will be pointing into the object.
    - The idea of equipotentials also transfers to the gravitational fields, the reason being that if an object is placed 10 m above the surface of the earth, then no work will have to be done to move the object to another place as long as it's 10 m above the surface.
        - This is because of the fact that the mass begins and ends its journey on the same equilateral point.
    - The contour lines (line on a map joining points of equal height) on a map represent the equipotential lines.
        - Walking on a contour line will mean that you will have the same vertical displacement, and therefore your gravitational potential energy won't change.
    - Just as for electric fields, a gravitational field is uniform and directed downward at 90° normal to the surface when close to the surface of the object.
- Worked example
    -





The field will be strongest between 10V and 20V equipotential lines because they're closer to one another compared to the other equipotential lines. Since the electric field strength is given by the equation: $E = \frac{kq}{r^2}$, the shorter the distance the stronger the field. Furthermore, work = distance × force, therefore Force = $\frac{\text{work}}{\text{distance}}$, showing that the smaller the distance the stronger the force.

$E = \frac{kq}{r^2}$

$E = \frac{(8.99 \cdot 10^{9})(1.6 \cdot 10^{-19})}{r^2}$

- Potential at a point
    - Gravitational potential
        - Gravitational potential difference is the work done in moving a unit mass (1 kg in our unit system) between two points.
        - To calculate the difference, we have to know the gravitational energy required to achieve the move, and the mass of the object.
            - $\Delta V_g = \frac{W}{m}$
                - ΔV_g is equal to the change in the gravitational potential, W is the work done, and m is the mass of the object.
                - ΔV_g units are $J\,kg^{-1}$
        - Gravitational potential is defined to be zero at infinity.
        - Newton's law of gravitation says that the force of attraction F between two point masses is inversely proportional.
            - $F \propto \frac{1}{r^2}$
        - At infinity we know that the gravitational force is zero due to the fact that the equation is $F \propto \frac{1}{r^2}$.
        - Energy has to done to the system to return the mass to infinity.
        - Whenever the masses are closer than infinity, the system of the two masses has a negative potential.
            - Negative potential potential means that work has to be done to bring the potential back to zero.
            - The closer the objects are to one another, the more negative the stored energy becomes because we have to put in more energy to put it back to infinite seperation.
            - At infinity, the gravitational potential is max and equal to zero.
                - At seperations less than infinity the masses have gravitational forces acting on each other, therefore they have negative gravitational potential.
        - The closer the they approach each other, the more negative the stored energy becomes because we have to put increasing amounts of energy back into the system to move the masses back to infinite distances.
        - Gravitational potential at a point is defined as: Equal to the work done per unit mass (kg) in moving a test mass from infinity to the point in question.
        - On Earth raising an object by one meter will result in an increase by $10\,J\,kg^{-1}$ for each meter raised.
        - To know the potential change we just have to know the mass of the object, and multiply it by the gravity.
            - $\Delta V_g = m \cdot g$
    - Electric potential
        - Just as gravitational potential the zero of potential is defined at infinity.
        - The change in electric potential ΔV_e is defined as:
            - ΔV_e = work done in moving charge between two points . W

magnitude of charge

- The energy stored in the system per unit charge is what we call the potential of the system.
- When two oppositely charged particles they will attract each other.
- When two charges are at a finite distance work will have to be done to bring the two charges to infinite distance.
    - As the charges are separated further and further, the potential will become closer and closer to zero.
- When two like charges are held at a finite distance, energy was stored in the system when the charges were brought together from infinity.
    - When the charges are released they will return to infinity where repulsive force is zero.
    - Opposite to opposite charges, when moving in a like charge closer to another like charge, work has to be done as they will repel one another. Therefore, the value for the potential will be above zero rather than below it.
- The electric potential at a point is the work done in bringing a unit positive test charge from infinity to the point.

- 10.2 Fields at Work
    - Equations:
        - Potential energy equations: $V_e = \frac{kQ}{r}$, $V_g = -\frac{GM}{r}$
        - Field strength equations: $E = -\frac{\Delta V_e}{\Delta r}$, $g = -\frac{\Delta V_g}{\Delta r}$
        - Relation between field strength and potential: $E_q = qV_e = \frac{kQq}{r}$, $E_p = mV_g = -\frac{GMm}{r}$
        - force laws: $F_E = k\frac{q_1 q_2}{r^2}$, $F_g = G\frac{m_1 m_2}{r^2}$
        - escape speed: $v_{esc} = \sqrt{\frac{2GM}{r}} = \sqrt{2gR}$
        - orbital speed: $v_{orbit} = \sqrt{\frac{GM}{r}}$ (m is the mass of the object
        - Gravitational field strength (Topic 6): $g = \frac{GM}{r^2}$
            - M = mass of source
    - Force and inverse-square law behaviour
        -

Electric fields	Gravity fields
Both fields obey an inverse-square law in which the force between two objects is inversely proportional to the distance between them squared.	
In a vacuum: $F_E = +\frac{kq_1 q_2}{r^2}$ where F_E is the force between two point charges q_1 and q_2 and r is the distance between them. This is known as Coulomb's law. The constant k in the equation is $\frac{1}{4\pi\varepsilon_0}$ where ε_0 is known as the permittivity of a vacuum or the permittivity of free space. If the field is in a medium other than a vacuum then the ε_0 in the equation is replaced by ε, the permittivity of the medium. The sign in the equation is positive. The sign of the overall result of a calculation indicates the direction in which the force acts. Negative indicates attraction between the charges; positive indicates repulsion.	$F_G = -\frac{Gm_1 m_2}{r^2}$ where F_G is the force between two point masses m_1 and m_2 and r is the distance between them. This is known as Newton's law of gravitation. The constant in the equation is G known as the universal gravitational constant. The value of G is a universal constant.

        - A positive force F_E indicates an attractive a repulsive force.
            - A negative one is attractive.
        - F_g is an attractive force.
    - Investigate
        - $Q \propto U$, $Q \propto A$, $Q \propto \frac{1}{d}$
            - $Q \propto \frac{UA}{d} \rightarrow Q = \varepsilon_0 \frac{UA}{d}$
                - $\varepsilon_0 = 8.854 \cdot 10^{-12}\ m^{-3} kg^{-1} s^4 A^2$, A = Area, d = distance between plates, Q = charge, U = voltage (pd).
    - Electric field strength and potential gradient
        - A positive charge that has size Q is in a field between two charged plates.
            - The field has strength E, and force F acts on charge.
            - $F = EQ \rightarrow$ Work done $= EQ \cdot x$ ($W = Fx$)
        - $E = \frac{U}{d}$ (electric field strength = $\frac{\text{potential}}{\text{distance}}$). $E = \frac{F}{Q}$
    - Worked example
        - a) $E = \frac{U}{d}$
          $= \frac{5000}{0.12}$
          $= 41667$
          $= 4.2 \cdot 10^4\ V m^{-1}$
        - b) $Q = 2 \cdot 1.6 \cdot 10^{-19}$
          $Q = 3.2 \cdot 10^{-19}$ J
          $F = EQ$
          $= (41666.67)(3.2 \cdot 10^{-19})$
          $F = 1.33 \cdot 10^{-14}$ N

Book rounds results before the final calculation like an idiot!!

        - 80 mm, $11.2 \cdot 10^{-19}$ C
            - $F = EQ$
            - $d = \underline{VQ}$

$F = \frac{UQ}{d}$

$U = \frac{(3.6 \cdot 10^{-14})(0.08)}{11.2 \cdot 10^{-19}}$

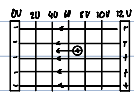
$U = 2571.43V$

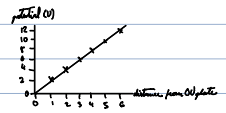
$V = 2.6kV$ ✓

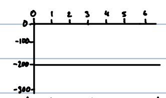
F

$= \frac{(2571.43)(11.2 \cdot 10^{-19})}{(1.4 \cdot 10^{-13})}$

$= \underline{20.57\,mm}$ ✓

- **Electric field strength and surface charge density**
  - The equation $Q = \frac{\varepsilon_0 UA}{d}$ can be rearranged to $\frac{Q}{A} = \varepsilon_0 \frac{U}{d}$.
    - $\frac{Q}{A} = \sigma$, which is the surface charge density on the plate.
    - E can be written as $\frac{U}{d}$.
  - Between the plates the electric field strength is: $E = \frac{\sigma}{\varepsilon_0}$.
    - This equation is for a field between two parallel plates.
  - Each plate contributes half the field, so the electric field E close to the the surface of any conductor is equal to:
    - $E = \frac{1}{2} \times \frac{\sigma}{\varepsilon_0} \rightarrow E = \frac{\sigma}{2\varepsilon_0}$
      - $\underline{\sigma}$ is the $\underline{\text{charge per unit area on the surface}}$ and ε_0 is the constant $8.854 \cdot 10^{-12}\,m^{-3}\,kg^{-1}\,s^4\,A^2$.
- **Graphical interpretations of electric field strength and potential**
  - 





^Electric field strength and potential
    - The equipotentials are equally spaced.
    - Since the electric field is uniform, the graph of potential from 0V to 12V against distance will be a straight line through the origin.
      - Giving E (electric field strength) = −potential gradient $= -\frac{\Delta V_e}{\Delta x}$
    - The negative sign in the equation $-\frac{\Delta V_e}{\Delta x}$ represents the direction of the vector electric field that its always opposite to the variations of the potential (ΔV_e) of the positive charge.
      - Therefore, moving in the opposite direction of the field will result in the gain in potential energy, and a positive sign.
        - moving from 0V to 12V = 12 − 0 = 12V = ΔV_e.
        - The equation is ΔV_e = final potential − initial potential.
    - What the equation simply shows that as you go from a high potential to a lower one, you'll lose potential energy.
    - Although, when an electron moves from 0V towards the 12V, its potential energy is reduced because because the potential energy change, ΔE_p, is given by $e\Delta V$. Since $e = -1.6 \cdot 10^{-19}$ J, then that means that the charges are opposite of a positive point charge.
      - If the electron is free to do so, then it will accelerate gaining kinetic energy from the field, with a loss of the electrical potential energy.
    - The equation $E = -\frac{\Delta V}{\Delta x}$ can be re-written as $E\Delta x = -\Delta V$.
    - Electric field strength is the force per unit charge.
    - Field strength equation: $E = \frac{kQ}{r^2} \rightarrow V_e = \frac{Q}{4\pi\varepsilon_0 r}$ or $\frac{kQ}{r}$ $\left(k = \frac{1}{4\pi\varepsilon_0}\right)$.
      - The equation predicts that the closer we are to a positive charge, the greater the potential is.
    - The work required to move a charge from infinity to the point where it will be is: $W = V_e q$.
    - If the charge q is in a field that arises from another single point charge Q then its potential energy $E_p = qV_e$ or $E_p = \frac{kqQ}{r} = \frac{qQ}{4\pi\varepsilon_0 r}$.
- **Worked example**
  - $E = \frac{V}{d}$
    $= \frac{1500}{0.25}$
    $= 6kV\,m^{-1}$
    $= 6kNC^{-1}$
  - −0.48 mC, 1.5m
    $V_e = \frac{Q}{4\pi\varepsilon_0 r} = \frac{kQ}{r}$
    $= \frac{(0.48 \cdot 10^{-9})(8.99 \cdot 10^{9})}{(1.5)}$
    $= 2.8768\,V$
    $= \underline{2.9V}$
- **Potential inside a hollow conducting charged sphere**
  - Outside of a charged conducting sphere, the field is identical to that of a point charge.
    - The reason being that the electric field lines leave the sphere at 90°.
  - **The field inside the sphere**
    - Since the field is a conductor, all the surplus charge must be outside of the sphere. This is because:
      - Charge will move until they are as far apart as possible
      - The charges will move until they're at equilibrium (when they're at equal distances to one another).
    - To find the field strength inside of the conductor, all of the forces acting on the charge have to be taken into consideration.
    - Forces on a positive test charge in a sphere.

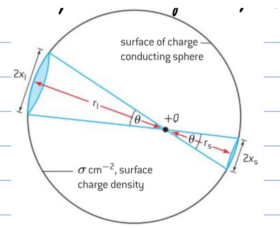


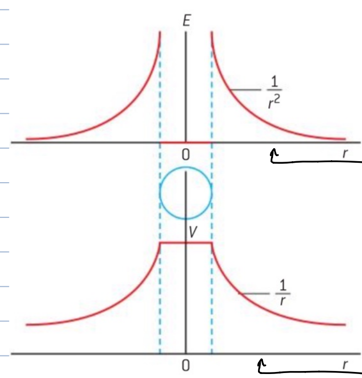
Figure 6 Field inside a charged conductor.

- As can be seen in the image, the cones have equal angles to one another.
  - The distance from the test charge to the sphere for the small cone is r_s.
  - The distance from the test charge to the sphere for the large cone is r_l.
  - The radiuses are instead x_s and x_l.
  - The surface charge density on the cone is σ, its the same for the whole sphere.
-

	For the small cone	For the large cone
Area of the end of the cone	πx_s^2	πx_l^2
Charge on the end of the cone	$\sigma\pi x_s^2$	$\sigma\pi x_l^2$
Distance of area from test charge	r_s	r_l
Force on test charge due to area	$\frac{kQ\sigma\pi x_s^2}{r_s^2}$	$\frac{kQ\sigma\pi x_l^2}{r_l^2}$
	Directed away from the surface	Directed away from the surface

- Since the test charge is the same, the forces will have to be equal to one another.
  - $\frac{kQ\sigma\pi x_s^2}{r_s^2} = \frac{kQ\sigma\pi x_l^2}{r_l^2}$
- Since they're equal to one another, they will cancel out giving 0 net force.
- Because the cones (representations of the electrical field strength) cancel one another out in all directions for all spheres, therefore there will be no electric field strength in the sphere, meaning no electric field in the sphere.
- No work is required to move a charge in the sphere as ΔV is 0.
  - The potential on the inside of the sphere is the same as the potential on the surface of the sphere.

-



Since the gradient is $\frac{1}{r^2}$, the equation is that of the electric field strength: $E = \frac{kQ}{r^2}$.

Since electrical potential is given by the equation: $E_p = \frac{kqQ}{r}$, this is the graph because the change is given by $\frac{1}{r}$.

Figure 7 Field and potential inside and outside the conductor.

- Gravitational potential
  - Gravitation
    - Similar to electrical fields strength where $E = -\frac{\Delta V_e}{\Delta x}$, for the gravitational field strength is: $g = -\frac{\Delta V_g}{\Delta r}$.
      - g represents the gravitational field strength, V_g is the gravitational potential, and r is the distance.
    - The potential V_g at r for a point mass is: $V_g = -\frac{GM}{r}$.
    - The potential energy of m at r from mass M is: $E_p = mV_g = -G\frac{Mm}{r}$. ($E_p = -\frac{Gm_1m_2}{r}$)
      - E_p is the potential energy stored, V_g is gravitational potential.
      - G is the gravitational constant, M is the mass, r is the distance, and m is the second point mass
    - For the gravity, the forces are always attractive.

- This results in the potential always being negative (as work has to be done to separate the masses and bring them back to infinite distance), and the maximum is zero at infinite distance.
- The gravitational potential ($E_{p(g)} = -\frac{Gm_1m_2}{r}$) is also a representation of the kinetic energy required to escape the gravitational pull of a mass.
  - Think of $E_{p(g)}$ as a potential well, and that a mass must have a required potential energy to escape the atmosphere.

- Worked example
  - $E_p = mV_g$ — Initial $= -8\,Jkg^{-1}$, Final $= -3\,Jkg^{-1}$, $\Delta V_g = 5\,Jkg^{-1}$

    $\Delta E_p = m\Delta V_g$

    $\Delta E_p = 20\,J$

  - $V_g = -\frac{GM}{R}$

    $= -\frac{(6.67\cdot10^{-11})(6\cdot10^{24})}{(6.4\cdot10^{6})}$

    $= -6.25\cdot10^{7}\,Jkg^{-1}$

- Potential inside a planet
  - We can think of Earth as having two parts, the solid sphere beneath our feet or, the shell "above our heads."
  - The inner solid sphere, with a smaller radius, behaves as a normal Earth but with a different gravitational field strength.
    - $g' = \frac{G(\text{reduced mass of Earth})}{(\text{reduced Earth radius})}$
    - The outer spherical shell doesn't have an effect on the gravitational field.
  - The reduced mass of the earth is equal to $\frac{4}{3}\pi\rho r'^3$.
    - ρ = density of the Earth so, $g' = \frac{4\pi G\rho r'}{3}$.
    - r' is the radius of the "small Earth".
  - The gravitational field is proportional to the distance from the center of the Earth until you reach the surface.

- Leaving the Earth
  - The velocity of a satellite in orbit is given by: $v_{orbit} = \sqrt{\frac{GM}{r}}$.
  - The orbital time T for the orbit is: $T_{orbit} = \sqrt{\frac{4\pi^2r^3}{GM}}$ (unit = seconds)
    - M is the mass of the planet and r is the radius of the orbit.
  - Two orbits are particularly important.
    - The polar orbit is used for satellites close to Earth's surface (100km from surface).
      - For the polar orbit, the linear speed ($v_{orbit} = \sqrt{\frac{GM}{r}}$) is $7800\,ms^{-1}$ or 28000km per hour.
      - Example: $T_{orbital} = \sqrt{\frac{4\pi^2r^3}{GM}}$

        $= \sqrt{\frac{4\pi^2(6.4\cdot10^6)^3}{(6.67\cdot10^{-11})(6\cdot10^{24})}}$ $6.5\cdot10^6$ is wrong (book)

        $T_{orbital} = 5085.24\,s$

        ≈ 5000 seconds

        $= 83.33$ min
    - A geostationary orbit is a special case of the geosynchronous orbit.
      - Geosynchronous satellites orbit at much greater distances from the Earth and have orbital times equal to one sidereal day which is roughly 24 hours.
      - A geostationary orbit is when a satellite is placed over the equator, and doesn't move.
  - Worked example
    - $24\cdot3600 = 86400$ $\quad 86400 = \sqrt{\frac{4\pi^2r^3}{GM}}$ $\quad 4.23\cdot10^7 - 6.4\cdot10^6 = 35900000\,m$ $= \underline{3.6\cdot10^7\,m}$

      $\sqrt[3]{\frac{(6.67\cdot10^{-11})(6\cdot10^{24})(86400^2)}{4\pi^2}} = r$

      $r = 4.23\cdot10^7\,m$

    - $g = \frac{GM_E}{(R_E+r)^2}$

      $= \frac{(6.67\cdot10^{-11})(6\cdot10^{24})}{(4.2\cdot10^7)^2}$

      $= 0.23\,N\,kg^{-1}$

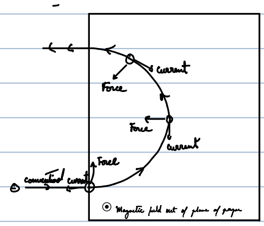
- Escaping the Earth
  - The total energy of a satellite is made up of the gravitational potential energy and the kinetic energy (ignoring energy transfer to the atmosphere).
    - To escape from the surface of the Earth, work has to be done (as gravity is always attractive (negative potential energy with 0 at infinity)) to bring the satellite and the Earth at infinite distances.
      - For an unpowered projectile this is simply equal to the kinetic energy. An unpowered projectile is something like a ball, it can't propel itself up.
        - This simply means that the gravitational potential energy and the kinetic energy must be equal to zero: gravitational potential energy + kinetic energy = 0.
        - $-\frac{GM_Em_s}{R_E} + \frac{1}{2}m_sv_E^2 = 0$

          - R_E is the distance of the satellite from the center of the Earth. v_E is the escape velocity. M_E = mass of earth. and m_s = mass of satellite.

- The gravitational potential energy is negative because it's a bound system. Kinetic energy is always positive.
- To escape the Earth's gravitational field completely the total energy of the satellite must be at least zero.
  - When the satellite is exactly zero (able to just reach infinity) then: $v_E = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2gR_E}$.
    - g is the gravitational field strength at the surface.
    - The speed to leave the surface of Earth needs to be about $11.2\,km\,s^{-1}$ ($40{,}000\,km\,h^{-1}$) to enter orbit. A higher speed than this will result in a positive speed at infinity.
- The escape velocity is the speed at which an unpowered object (e.g bullet) would have to be travelling at to leave Earth's surface.
  - Generally: Escape Velocity is the minimum speed needed for a free, non-propelled object to escape from the gravitational influence of a massive body.
  - Escape velocity is only required to send a ballistic object on a trajectory that will allow the object to escape the gravity well of the mass M (Earth).
  - In theory a rocket can escape Earth's gravitational field at any velocity as long as it has the required fuel.
    - It must simply supply 63MJ for each kilogram of the mass of the rocket (at Earth's surface the gravitational potential is $-62.5\,MJ\,kg^{-1}$).
- Before takeoff, a spaceship sits in a potential well on the Earth's surface.
  - It needs enough energy to reach the max at the potential at Lagrangian point (where gravitational field strength of Earth & the moon are equal). Once the space ship is at that point it can fall down to arrive at the Moon.

- Orbit shapes
  - Orbital speed close to the surface is $7.9\,km\,s^{-1}$, and the escape speed is $11.2\,km\,s^{-1}$.
    - At speeds between these values the spaceshift will achieve different orbits.
      - Speeds less than $7.9\,km\,s^{-1}$ the spaceshift will return back to Earth.
      - Speeds of $7.9\,km\,s^{-1}$ the spaceship will have a circular path.
      - Speeds between $7.9\,km\,s^{-1}$ and $11.2\,km\,s^{-1}$ the orbit will be an ellipse.
      - Speeds of $11.2\,km\,s^{-1}$ the craft will escape in a parabolic trajectory.
      - Speeds greater than $11.2\,km\,s^{-1}$ the craft will escape the Earth along a hyperbolic path.

- Charges moving in magnetic and electric fields
  - Charges moving in magnetic fields
    - In the figure below an electron moving to the right will be affected by the magnetic field at 90° to it.



The diagram shows the Fleming left hand rule (where the thumb is the force, the index finger is the field, and the middle finger is the current).

- Fleming's left-hand rule states that the effect on the electron.
  - The force will be acting on the electron at 90° to the conventional current, pushing the electron.
    - The current is backwards because it's conventional current.
- The electron accelerates in response to the force, and its direction of motion must change.
  - The direction of this change is such that it will still travel at 90° to the field and the magnetic force will continue to be at right angles to the electron's new direction.
  - Exact requirements for an electron to move in a circle.
    - The magnetic force (force on the diagram) is what provides the centripetal force, which changes direction, showing that the electron is accelerating towards the center.
- The force acting on the electron depends on its charge q, its speed v, and the magnetic field strength B.
  - The centripetal force will lead to a circular orbit of radius r and:
    - $\frac{m_e v^2}{r} = Bqv$
- The radius of the circle is:
  - $r = \frac{m_e v}{Bq} = \frac{p}{Bq} = \frac{(2m_e E_k)^{\frac{1}{2}}}{Bq}$
    - p is momentum, m_e is the mass of the electron.
- To determine the specific charge on the electron (charge per unit mass ($C\,kg^{-1}$)).
  - A fine-beam tube is used in which a beam of electrons is fired through a gas at very low pressures (meaning that the electron won't collide with too many gas atoms).

- when a uniform magnetic field is applied at right angles to the beam direction, the electrons move in a circle and their path is shown by the emission of visible light from atoms excited by the collisions with electrons along the path.

- the electrons of mass m_e are accelerated using a potential difference U before entering the field.
  - $\frac{1}{2} m_e v^2 = qU$ so $v = \sqrt{\frac{2qU}{m_e}}$ which with $v = \frac{Bqr}{m_e}$ gives $\frac{q}{m_e} = \frac{2U}{B^2 r^2}$.

- When an electron beam is moving into a magnetic field (not 90° to the beam direction) then the perpendicular electron velocity will lead to a circular motion.
  - the difference is that: $r = \frac{m_e v \sin\theta}{Bq}$
    - where θ is the angle between the field direction and the beam direction.
    - the radius of this case will be smaller than the perpendicular case.
  - the parallel velocity does not lead to a circular motion.
    - the electrons will move at the component speed ($v\cos\theta$) in a helical path.

Summary

- To summarise, when a charged particle (e.g. electron) moves through a magnetic field where the electron moves to the right of a uniform magnetic field where the field lines are oriented at 90° to the direction which the electron moves (force, 90°, field lines, current (non-conventional)) this will mean that the electron will accelerate towards the center (due to force), meaning that its direction of motion will change. This will mean that the electrons will in in circular motion (as the direction its travelling in & the force will be at constant 90° angles).
  - the circular motion will also alter the direction of the force.
  - the force is calculated by: $Bqv = \frac{m_e v^2}{r}$.
  - the speed of the electrons can be calculated by: $v = \sqrt{\frac{2qU}{m_e}}$.
  - the radius of the circle is: $r = \frac{m_e v}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2 m_e E_k}}{Bq}$
- For charged particles moving in a field which isn't at 90° to the direction of the charged particle, then the particles perpendicular and parallel velocities must be considered.
  - the particle will still be in circular motion in the perpendicular direction, but the radius will change to: $r = \frac{m_e v \sin\theta}{Bq}$
    - θ is the angle between the field direction and the beam direction.
      - Radius of the circle will be less than the perpendicular.
  - the particle won't be in circular motion in the parallel component, the electrons will move at the component speed ($v\cos\theta$) resulting in a helical path.

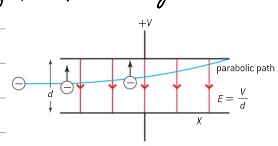
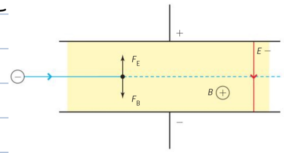
- Charges moving in electrical field
  - this changes for a particle moving in an electric field.
  - 

Figure 16 Motion of an electron in a uniform electric field.

- the force on the electrons now acts parallel to the field lines.
  - In the figure above the particle is accelerating upwards.
    - Since the force's magnitude and direction is constant the acceleration is also constant.
    - Acceleration is given by: $a = \frac{F}{m_e} = \frac{qE}{m_e} = \frac{qU}{m_e d}$
- Horizontal velocity is constant.
- Time to travel between two plates is: $t = \frac{x}{v_{horizontal}}$.
- Vertical component is: $v_{vertical} = a_{vertical} \cdot t = \left(\frac{qU}{m_e d}\right)\left(\frac{x}{v_{horizontal}}\right)$
- The final speed (leaving plates) is $v = \sqrt{(v_{vertical}^2 + v^2_{horizontal})}$
- the deflection (s) of the particle is given by the equation: $s = \frac{1}{2} a t^2$
  - the deflection is also proportional to t^2.
  - this means that the trajectory of the electron is parabolic not circular.

- Charge moving in magnetic and electric fields
  - When magnetic fields and electric fields are combined at right angles to each other, the electric force is vertically upwards, and the magnetic force is downwards.
    - When the forces are equal, the net force is zero on the charged particle.

- $F_E = F_B \quad qE = Bqv \longrightarrow v = \frac{E}{B}$



F_E = Electric Force

F_B = magnetic force

Figure 17 Crossed electric and magnetic fields can cancel out.

- the E:B ratio shows that there is only one speed where the forces are balanced.
  - charged particles travelling more slowly than this speed will have a larger electric force than magnetic force, the particle will be accelerating upwards.
  - a faster particle will mean that the magnetic force will be greater than the electric force, and will be accelerating downwards.
- this arrangement is known as a velocity selector.

- Graphs and Tables

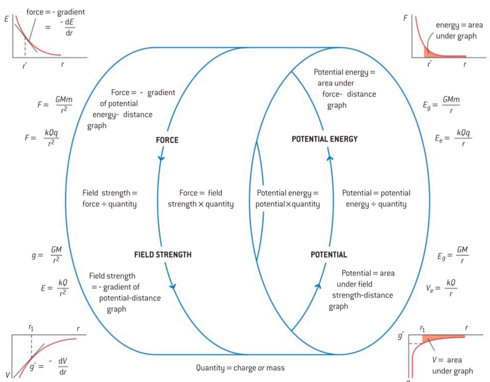


Figure 19 Relationships between field quantities.

		Electric	Gravitational
Force law		$F = \frac{Qq}{4\pi\varepsilon_0 r^2}$ (Coulomb's law)	$F = \frac{GMm}{r^2}$ (Newton's law)
Field strength	Modification when not in a vacuum	Replace ε_0 with ε	No change
	Definition	$E = \frac{F}{q}$	$g = \frac{F}{m}$
	Unit	N C^{-1} or V m^{-1}	N kg^{-1} or m s^{-2}
	Distance r from a point object	$E = \frac{Q}{4\pi\varepsilon_0 r^2}$	$g = \frac{GM}{r^2}$
	At r from centre of sphere of radius $R, r \geq R$	$E = \frac{Q}{4\pi\varepsilon_0 r^2}$	$g' = \frac{GM}{r^2}$
	At r from centre of sphere of radius $R, r < R$	$E = 0$	$g' = \frac{4\pi G\rho r'}{3}$
Potential	Definition	Electric potential energy per unit charge	Gravitational potential energy per unit mass
	Unit	$\text{V} \equiv \text{J C}^{-1}$	J kg^{-1}
	For two point charges or masses	$V_p = \frac{Q}{4\pi\varepsilon_0 r}$	$V_g = -\frac{GM}{r}$
Differences value at infinity: Attractive: zero and maximum Repulsive: zero and minimum		• Between charges • Attractive and repulsive depending on charge sign	• Between masses • Only attractive
	Constant in force law	$\frac{1}{4\pi\varepsilon_0}$ (= Coulomb constant k)	G